

Supplementary Information for

Regularized Machine Learning Models for Prediction of Metabolic Syndrome Using *GCKR*, *APOA5*, and *BUD13* Gene Variants: Tehran Cardiometabolic Genetic Study

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Regularized machine learning models

Consider the general linear regression model

$$y_i = b_0 + \sum_{j=1}^k b_j x_{ij} + \varepsilon_j$$

where y_i ($i=1, \dots, n$; n is the sample size) is the i th observation of the response variable, x_{ij} is the i th observation of the j th covariate ($(j=1, \dots, k)$, b_j 's), are the regression coefficients, ε_j are i.i.d. random error terms which $\varepsilon_j \sim N(0, \sigma^2)$. The ordinary least squares estimator of $b=(b_0, b_1, \dots, b_p)'$, is obtained by minimizing the residual sum of squares (RSS), i.e.,

$$b_{ols} = \underset{b}{\operatorname{argmin}} \sum_{i=1}^n (y_i - b_0 - \sum_{j=1}^k b_j x_{ij})^2,$$

Regularized machine learning (RML) models avoid the overfitting problem by penalizing the model complexity and adding a nonnegative regularization term to the log-likelihood function and, consequently, shrink the values of regression coefficients (1, 2).

$$\text{RML} = \text{RSS} + \lambda \omega(b)$$

Where $\lambda \omega(b)$ is the regularization term. As the amount of shrinkage is controlled by the regularization parameter $\lambda \geq 0$, choosing λ is an essential part of model fitting. A larger value of λ , leads to a greater amount of shrinkage. This study used a 10-fold Cross-validation method to select the optimal λ value for our models. Considering several regularization terms that have been previously proposed (1, 3-6), we applied five popular regularized machine learning models to select relevant features for MetS predicting models include the least absolute shrinkage and selection operator (LASSO) (1), ridge regression (RR) (3), elastic net (ENET) (6), adaptive LASSO (aLASSO) (5), and adaptive elastic-net (aENET). Supplementary table 1 shows the penalty factors and features of applied methods in the present study.

Received: 25/April/2023, Revised: 08/June/2023, Accepted: 19/June/2023

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Table S1: Penalty factors and features of regularized machine learning methods

Models	λ Penalty factor ($\omega(\mathbf{b})$)	Features
LASSO	$\lambda \sum_{j=1}^k b_j $	Perform variable selection by shrinking some predictor coefficients to be exactly zero. Improve the interpretability of the model. Does not capture any grouping effect. May fail when there is a strong correlation between predictors.
Ridge	$\lambda \sum_{j=1}^k b_j^2$	Shrinks the coefficients' estimates towards zero but does not force them to be exactly zero. Does not perform variable selection, meaning all features are retained even if they are not important. Helps in reducing the impact of multicollinearity among predictors. Works well when there are many relevant features.
ENET	$\lambda_1 \sum_{j=1}^k b_j + \lambda_2 \sum_{j=1}^k b_j^2$	Combines the strengths of LASSO and ridge regression. Simultaneously perform variable selection and shrink coefficients of correlated variables. Works well when there are many relevant features and some of them are highly correlated. Can handle correlated predictors better than LASSO.
aLASSO	$\lambda \sum_{j=1}^k w_j b_j $	Have all the good properties of the lasso. Simultaneously estimate and variable selection. Has the oracle properties. This method used adaptive weights for penalizing different coefficients to overcome the inconsistency of LASSO.
aENET	$\lambda_1 \sum_{j=1}^k w_j b_j + \lambda_2 \sum_{j=1}^k w_j b_j^2$	Has the oracle properties. Can handle the collinearity. Combines the advantages of both elastic net and adaptive LASSO.

LASSO; Least absolute shrinkage and selection operator, ENET; Elastic Net, aLASSO; Adaptive LASSO, and aENET; Adaptive Elastic Net.

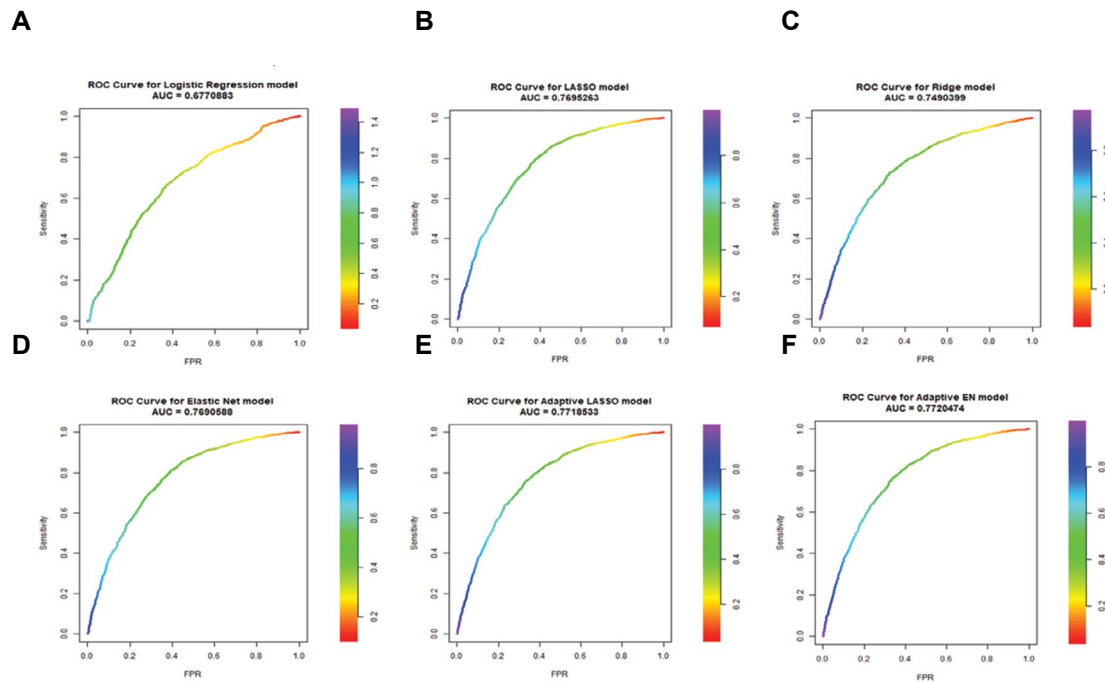


Fig.S1: ROC curves for penalized and logistic regression methods. **A.** ROC curve for logistic regression, **B.** ROC curve for LASSO regression, **C.** ROC curve for ridge regression, **D.** ROC curve for elastic net model, **E.** ROC curve for adaptive lasso model, **F.** ROC curve for adaptive elastic net model; ROC; Receiver operating characteristics, AUC-ROC; Area under the ROC curve, LASSO; Least absolute shrinkage and selection operator, EN; Elastic net, and FPR; False positive rate. The adaptive EN model shows a higher AUC-ROC curve.

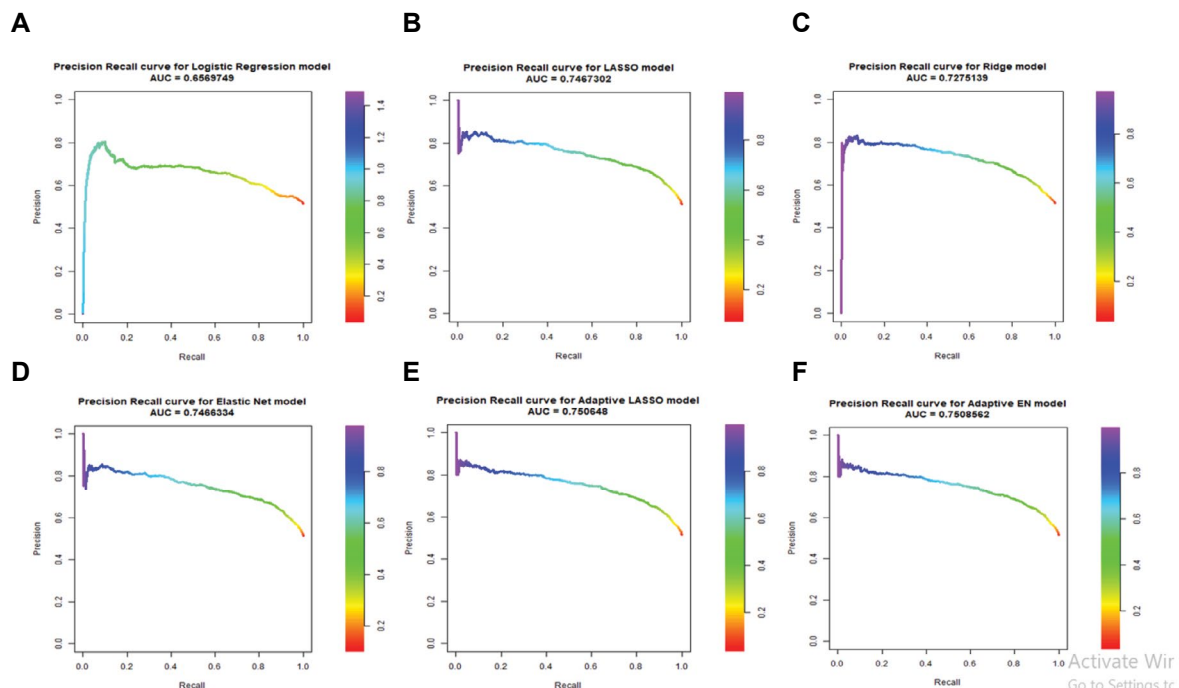


Fig.S2: Precision-recall (PR) curves for penalized and logistic regression methods. **A.** PR curve for logistic regression, **B.** PR curve for LASSO regression, **C.** PR curve for ridge regression, **D.** PR curve for elastic net model, **E.** PR curve for adaptive lasso model, **F.** PR curve for adaptive elastic net model. AUC-PR; Area under the precision-recall curve, LASSO; Least absolute shrinkage and selection operator, EN; Elastic net, and FPR; False positive rate. The adaptive EN model shows a higher AUC-PR curve.

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